LESSON #17 - FACTORING
COMMON CORE ALGEBRA II

In the study of algebra there are certain skills that are called “gateway skills” because without them a student simply cannot enter into many more complex and interesting problems. Perhaps the most important gateway skill is that of factoring. The definition of factor, in two forms, is given below.

**FACTOR – TWO IMPORTANT MEANINGS**

(1) **Factor (verb)** – To rewrite a quantity as an equivalent product.
(2) **Factor (noun)** – Any individual component of a product.

You should be familiar with factoring integers as well as algebraic expressions from earlier courses. We will review some of the basic concepts and techniques of factoring in this lesson.

**Exercise #1:** Factor each of the following integers completely. In other words, write them as the product of only prime numbers (called prime factorization).

(a) 12  
(b) 30  
(c) 16  
(d) 36

Always keep in mind that when we **factor (verb)** a quantity, we are simply rewriting it in an different form that is completely equal to the original quantity. It might look different, but \(2 \cdot 3\) is still the number 6.

**Exercise #2:** Rewrite each of the following binomials as a product of an integer with a different binomial.

(a) \(5x + 10\)  
(b) \(2x - 6\)  
(c) \(6x + 15\)  
(d) \(6 - 14x\)

The above type of factoring is often referred to as “factoring out” the greatest common factor (gcf). This greatest common factor can be comprised of numbers, variables, or both.

**Exercise #3:** Factor each of the following expressions:

(a) \(3x^2 + 6x\)  
(b) \(20x - 5x^2\)  
(c) \(10x^2 + 25x\)  
(d) \(30x^2 - 20\)

**Exercise #4:** Rewritten in factored form \(20x^2 - 36x\) is equivalent to

(1) \(2x(10x - 15)\)  
(2) \(4x(5x - 9)\)  
(3) \(5x(4x + 7)\)  
(4) \(9x(x - 4)\)
Trinomials can also sometimes be factored into the product of a gcf and another trinomial.

**Exercise #5:** Rewrite each of the following trinomials as the product of its gcf and another trinomial.

(a) \(2x^2 + 8x + 10\)  
(b) \(10x^2 - 20x + 5\)  
(c) \(8x^3 - 12x^2 + 20x\)  
(d) \(6x^3 + 15x^2 - 21x\)

Another type of factoring that you should be familiar with involves the conjugate multiplication pattern. This can be “reversed” in order to factor binomials that have the form of the difference of perfect squares.

**Exercise #6:** Factor each of the following expressions:

(a) \(x^2 - 9\)  
(b) \(4 - x^2\)  
(c) \(4x^2 - 25\)  
(d) \(16 - 81x^2\)

**Exercise #7:** Write each of the following binomials as the product of a conjugate pair.

(a) \(x^2 - \frac{1}{4}\)  
(b) \(25 - \frac{x^2}{9}\)  
(c) \(\frac{4}{81}x^2 - \frac{49}{9}\)  
(d) \(36x^2 - 49y^2\)

Factoring an expression until it cannot be factored anymore is known as **complete factoring**. Complete factoring is an important skill to master in order to solve a variety of problems. In general, when completely factoring an expression, the **first** type of factoring always to consider is that of factoring out the gcf.

**Exercise #8:** Using a combination of gcf and difference of perfect squares factoring, write each of the following in its completely factored form.

(a) \(5x^2 - 20\)  
(b) \(28x^2 - 7\)  
(c) \(40 - 250x^2\)
**Exercise #9:** The volume of a particular rectangular box is given by the equation \( V = 3x^3 - 48x \). The height and length of the box are shown on the diagram below. Find the width of the box in terms of \( x \). Recall that \( V = L \cdot W \cdot H \) for a rectangular box.

![Diagram of rectangular box](image)

Just as there is a pattern for factoring the difference of perfect squares, there are formulas for factoring the sum AND difference of perfect cubes.

**Exercise #10:** To verify the sum of perfect cubes formula, simplify the following product:

\[(a + b)(a^2 - ab + b^2)\]

**Exercise #11:** Factor each of the following expressions:

(a) \( x^3 - 8 \)
Let \( a = \)
Let \( b = \)

(b) \( 27x^3 + 1 \)
Let \( a = \)
Let \( b = \)

(c) \( x^3y^3 - 64 \)
Let \( a = \)
Let \( b = \)

(d) \( 125x^6 + 8 \)
Let \( a = \)
Let \( b = \)

(e) \( 5x^3 + 40 \)
Let \( a = \)
Let \( b = \)

(f) \( 16x^3 - 54 \)
Let \( a = \)
Let \( b = \)
LESSON #17 - FACTORING
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Rewrite each of the following binomials as the product of an integer with a different binomial.
   (a) $10x - 55$  
   (b) $24x - 40$  
   (c) $6x - 45$  
   (d) $18x - 9$

2. Factor each of the following expressions:
   (a) $2x^2 - 8x$  
   (b) $6x + 27$  
   (c) $30x^2 - 35x$  
   (d) $24x^3 + 20x^2$

3. Rewrite each of the following binomials as the product of a conjugate pair.
   (a) $x^2 - 121$  
   (b) $64 - x^2$  
   (c) $4x^2 - 1$  
   (d) $25x^2 - \frac{1}{9}$

4. Factor each of the following expressions:
   (a) $4x^2 + 12x + 28$  
   (b) $6x^2 - 4x + 10$  
   (c) $14x^3 + 35x^2 - 7x$  
   (d) $20x^3 - 5x^2 + 15x$

5. Completely factor each of the following binomials using a combination of gcf factoring and conjugate pairs.
   (a) $6x^3 - 150$  
   (b) $36 - 4x^2$  
   (c) $28x^2 - 7$  
   (d) $27x^3 - 12x$

   (e) $80 - 125x^2$  
   (f) $2x^3 - 200x$  
   (g) $8x^2 - 512$  
   (h) $44x - 99x^3$
6. Factor each of the following expressions:

(a) \( x^3 + 27 \)  
(b) \( 8x^3 - 216 \)  
(c) \( x^6 - 8 \)

7. When completely factored, the expression \( 48 - 3x^2 \) is written as

1. \( 3(16 - x)(16 + x) \) 
2. \( 3(x - 16)(x + 16) \) 
3. \( 3(x - 4)(x + 4) \) 
4. \( 3(4 - x)(4 + x) \)

8. Which of the following represents the greatest common factor of the terms \( 4x^2y^6 \) and \( 18xy^5 \)?

1. \( 36xy \) 
2. \( 4x^2y^3 \) 
3. \( 2xy^5 \) 
4. \( 2x^2y^2 \)

9. Which of the following is not a factor of \( 6x^2 - 18x \)?

1. \( x - 3 \) 
2. \( 2 \) 
3. \( 12 \) 
4. \( x \)

10. Which of the following prime numbers is not a factor of the integer 330?

1. \( 11 \) 
2. \( 7 \) 
3. \( 3 \) 
4. \( 5 \)

APPLICATIONS

11. The area of any rectangular shape is given by the product of its width and length. If the area of a particular rectangular garden is given by \( A = 15x^2 - 35x \) and its width is given by \( 5x \), then find an expression for the garden’s length. Justify your response.
12. The volume of a particular rectangular box is given by the equation \( V = 50x - 2x^3 \). The height and length of the box are shown on the diagram below. Find the width of the box in terms of \( x \). Recall that \( V = L \cdot W \cdot H \) for a rectangular box.

![Diagram of a rectangular box with dimensions labeled]

13. A projectile is fired from ground level such that its height, \( h \), as a function of time, \( t \), is given by \( h = -16t^2 + 80t \). Written in factored form this equation is equivalent to

(1) \( h = -16t(t + 4) \)  
(2) \( h = -8t(2t - 7) \)  
(3) \( h = -16t(t - 5) \)  
(4) \( h = -8t(t - 5) \)
LESSON #18 - FACTORING BY GROUPING
COMMON CORE ALGEBRA II

You now have essentially three types of factoring: (1) greatest common factor, (2) difference of perfect squares, and (3) sum and difference perfect cubes. Today we will introduce a new type of factoring known as factoring by grouping. This technique requires you to see structure in expressions.

Exercise #1: Factor a binomial common factor out of each of the following expressions. Write your final expression as the product of two binomials.

(a) \( x(2x+1)+7(2x+1) \)
(b) \( 5x(x-2)-4(x-2) \)
(c) \( (x+5)(x-7)+(x-7)(x+1) \)
(d) \( (2x+8)(x+4)-(x-2)(x+4) \)

Exercise #2: Write the expression \((x+3)(x-4)+5(x+3)\) as the equivalent product of binomials. Test this equivalency with \(x = 2\).

Some very special polynomials can be factored by taking advantage of the structure we have seen in the last two problems. The key is to do mindful manipulations of expressions so that they remain equivalent but are written as an overall product.

Exercise #3: Consider the expression \(2x^3 - 6x^2 + 5x - 15\). Justify each step below with one of the three major properties of real numbers, i.e. the commutative, associative, or distributive.

\[
2x^3 - 6x^2 + 5x - 15 = (2x^3 - 6x^2) + (5x - 15) = 2x^2(x - 3) + 5(x - 3) = (x - 3)(2x^2 + 5)
\]
When we **factor by grouping** we first extract common factors from pairs of binomials in four-term polynomials. If we are **lucky** we are left with another **binomial common factor**.

**Exercise #4:** Use the method of factoring by grouping to completely factor the following expressions.

(a) $6x^2 - 4x + 15x - 10$

(b) $2x^2 - 30x + x + 15$

(c) $5x^3 - 2x^2 + 5x - 2$

(d) $18x^3 + 9x^2 - 2x - 1$

(e) $3x^3 + 2x^2 - 27x - 18$

(f) $x^2y + 3xy - 25x - 75$

(g) $x^5 + 4x^3 + 2x^2 + 8$

(h) $5x^3 + 10x^2 + 20x + 40$
**Exercise #5:** Consider the expression \( x^2 + ab - ax - bx \).

(a) How can you rewrite the expression so that the first two terms share a common factor (other than 1)?

(b) Write this expression as an equivalent product of binomials.

Be careful when you use factoring by grouping. Don't force the method when it does not apply. This can lead to errors.

**Exercise #6:** Consider the expression \( 2x^3 + 10x^2 + 7x + 21 \). Explain the error made in factoring it. How can you tell that the factoring is incorrect?

\[
2x^3 + 10x^2 + 7x + 21 = 2x^2(x + 5) + 7(x + 3) \\
= (2x^2 + 7)(x + 5 + x + 3) \\
= (2x^2 + 7)(2x + 8)
\]
Lesson #18 - Factoring by Grouping
Common Core Algebra I Homework

Fluency

1. Rewrite each of the following as the product of binomials. Be especially careful on the manipulations that involve subtraction.

   (a) \( x(x+5)+7(x+5) \)
   
   (b) \( 4x(x-2)-3(x-2) \)

   (c) \( (x+10)(x-3)+(x+5)(x-3) \)
   
   (d) \( (2x-7)(x+4)+(x+4)(x+2) \)

   (e) \( (4x+3)(2x-1)-(x+2)(2x-1) \)
   
   (f) \( (3x-7)(x+5)-(x+5)(2x-4) \)

2. Max tries to simplify the expression \( (5x+2)(x+3)-(2x-3)(x+3) \) as follows:

   \[
   = (5x+2)(x+3)-(2x-3)(x+3) \\
   = (x+3)(5x+2-2x-3) \\
   = (x+3)(3x-1)
   \]

   Show using \( x=2 \) that this simplification is incorrect. Then, give the correct simplification.
3. Completely factor each of the following expressions:
   (a) \(10x^2 + 6x + 35x + 21\)  
   (b) \(12x^2 + 3x - 20x - 5\)

   (c) \(5x^3 + 2x^2 - 20x - 8\)  
   (d) \(18x^3 - 27x^2 - 2x + 3\)

   (e) \(x^3 + 2x^2 - 25x - 50\)  
   (f) \(8x^3 + 10x^2 + 12x + 15\)

4. Factor each of the following expressions. Rearrange the expressions as needed to produce binomial pairs with common factors.
   (a) \(x^2 - ac - cx + ax\)
   (b) \(xy + ab + ay + bx\)

**REASONING**

5. Consider the expression: \(x^3 - 5x^2 - 9x + 45\). Enter this expression in \(y=\) on your calculator and find its zeroes (x-intercepts). Use the following window: \(Xmin: -10, Xmax: 10, Ymin: -10, Ymax: 50\). Draw a rough sketch. Then, factor the expression completely. Do you see the relationship between the factors and the zeroes?
Factoring trinomials, expressions of the form \( ax^2 + bx + c \), is an important skill. Trinomials can be factored if they are the product of two binomials. The two main keys to factoring trinomials are: (1) the ability to quickly and accurately multiply binomials (FOIL) and (2) the ability to work with signed numbers. We practice both of these skills with three warm-up multiplication problems in Exercise #1.

**Exercise #1:** Without using your calculator, write each of the following products in simplest \( ax^2 + bx + c \) form.

(a) \((3x + 2)(5x + 7)\)  
(b) \((5x - 4)(x - 2)\)  
(c) \((4x + 3)(3x - 8)\)

**Exercise #2:** Consider the trinomial \(6x^2 - 35x - 6\). Below are four guesses of how this trinomial factors.

\((3x + 2)(2x - 3)\)  
\((x - 3)(x + 2)\)  
\((6x + 1)(x - 6)\)  
\((3x - 2)(2x - 3)\)

(a) Two of these guesses are “unintelligent” – meaning that they should not even be checked. Cross them out and explain below them why they are unreasonable.

(b) Of the two that remain, check both above and determine which is the correct factorization of the trinomial.

The easiest of all trinomial factoring occurs when the leading coefficient is one \((a = 1)\).

**Exercise #3:** Using a guess-and-check technique, factor each of the following trinomials.

(a) \(x^2 + 2x - 35\)  
(b) \(x^2 + 11x + 24\)  
(c) \(x^2 - 13x + 22\)  
(d) \(x^2 - 5x - 50\)

(d) \(x^2 - 11x - 12\)  
(e) \(x^2 - 10x + 21\)  
(f) \(x^2 - 16x + 48\)  
(g) \(x^2 - x - 72\)
When the leading coefficient is not one, we can use the technique of grouping to factor trinomials.

- Find two factors whose sum is \(b\) and whose product is \(ac\)
- Express the middle term as the sum of these two factors. (It does not matter which factor you write first).
- Now that you have four terms, use grouping to factor the expression:
  - Use GCF factoring on the first two terms and the last two terms.
  - Factor out the common binomial factor

<table>
<thead>
<tr>
<th>Trinomial</th>
<th>Factored Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2x^2 + 5x - 18)</td>
<td>(9 + 4 = 5)</td>
</tr>
<tr>
<td>(2x^2 + 9x - 4x - 18)</td>
<td>(9(-4) = -36)</td>
</tr>
<tr>
<td>(x(2x + 9) - 2(2x + 9))</td>
<td>(2x + 9)(x - 2)</td>
</tr>
</tbody>
</table>

**Exercise #3:** Use the method of grouping to determine the correct factorization of the following trinomials.

(a) \(4y^2 - 9y + 2\)  
(b) \(6x^2 - x - 2\)  
(c) \(3x^2 + 19x - 40\)  
(d) \(2x^2 - 15x + 18\)  
(e) \(15x^2 + 13x + 2\)  
(f) \(10x^2 + 13x - 30\)  
(g) \(12x^2 + 8x - 15\)  
(h) \(36x^2 - 35x + 6\)
LESSON #19 - FACTORING TRINOMIALS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Multiply each of the following binomial pairs and express your answer in simplest trinomial form.
   
   (a) \((2x+5)(3x-2)\)
   
   (b) \((3x-8)(5x-1)\)

2. Which of the following is the correct factorization of the trinomial \(12x^2-23x+10\)? Hint – eliminate two of the choices because they are “unintelligent” guesses.

   (1) \((6x-1)(3x-10)\)
   
   (2) \((6x-2)(2x-5)\)
   
   (3) \((4x-5)(3x+2)\)
   
   (4) \((4x-5)(3x-2)\)

3. Written in factored form \(x^2+16x-36\) is equivalent to

   (1) \((x-3)(x+12)\)
   
   (2) \((x-6)(x+6)\)
   
   (3) \((x-2)(x+18)\)
   
   (4) \((x-9)(x+4)\)

4. Write each of the following trinomials in its factored form. These are the easiest trinomials to factor because the leading coefficient is equal to one.

   (a) \(x^2-7x-18\)
   
   (b) \(x^2+14x+24\)
   
   (c) \(x^2-17x+30\)
   
   (d) \(x^2-5x-6\)

   (e) \(x^2-5x+6\)
   
   (f) \(x^2-15x+44\)
   
   (g) \(x^2+21x+20\)
   
   (h) \(x^2-6x-16\)
5. Factor each of the following trinomials.

(a) $2x^2 \quad 5x \quad 3$

(b) $5x^2 \quad 7x \quad + 2$

(b) $5x^2 - 41x + 8$

(c) $3x^2 + 4x - 20$

(d) $2x^2 - 29x - 15$

(e) $7x^2 + 39x + 20$

(f) $18x^2 - 25x + 8$

(g) $20x^2 - 11x - 42$
LESSON #20 - COMPLETE FACTORING
COMMON CORE ALGEBRA II

Each expression that we have factored has been the product of two quantities. But, factoring can produce many more than just two factors. In Exercise #1, we first warm-up by multiplying three factors together.

Exercise #1: Write each of these in their simplest form. The second question should take little time to do.

(a) \(2(x+4)(x+7)\)  \hspace{1cm}  (b) \(4x(3x-2)(3x+2)\)

To completely factor an expression means to write it as a product which includes binomials that contain no greatest common factors (gcf’s).

Exercise #2: Consider the trinomial \(2x^2-4x-6\).

(a) Verify that both of the following products are correct factorizations of this trinomial.
\[
(2x-6)(x+1) \quad (2x+2)(x-3)
\]

(b) Why are neither of these completely factored?

(c) Write each of these in completely factored form by factoring out the gcf of each unfactored binomial.

(d) What is true of both complete factorizations you found in part (c)?

In practicality, it is always easiest to completely factor by looking for a gcf of the expression first. Once removed, the factoring then either consists of the difference of perfect squares or standard trinomial techniques.

Exercise #3: Write each of the following in its completely factored form. These should be relatively easy.

(a) \(4x^2 + 12x - 40\)  \hspace{1cm}  (b) \(6x^2 - 24\)  \hspace{1cm}  (c) \(2x^2 + 20x + 50\)  \hspace{1cm}  (d) \(75 - 3x^2\)
Exercise #4: Completely factor each of the following expressions. There may or may not be two steps in the factorization of these expressions.

(a) $10x^2 + 55x - 105$
(b) $12x^2 + 57x - 15$

(c) $14x^2 - 21x - 35$
(d) $12x^2 - 14x - 6$

(e) $6x^2 - 13x + 6$
(f) $3x^3 + 2x^2 - 12x - 8$

(g) $a^2 + 3ab + 2a + 6b$
(h) $12x^2 + 29x - 8$

(i) $y^3 - 4y$
(j) $x^3 + 2x^2 - 63x$
LESSON #20 - COMPLETE FACTORING
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Find each of the following products in their simplest $ax^2 + bx + c$ form.
   (a) $5(x - 6)(x - 2)$
   (b) $3(2x - 1)(2x + 1)$

2. Write each of the following expressions in their completely factored form.
   (a) $2x^2 - 14x - 36$
   (b) $5x^2 + 70x + 245$
   (c) $3x^2 - 192$
   (d) $6x^3 + 36x^2 - 96x$
   (e) $28x - 7x^3$
   (f) $8x^2 + 12x - 8$
   (g) $15x^2 - 110x + 120$
   (h) $10x^3 - 26x^2 - 12x$
   (i) $8x^2 + 67x + 24$
   (j) $12x^2 - 20x + 3$
(k) \(18x^2 - 39x - 15\)  

(l) \(45x - 20x^3\)

(m) \(7x^3 - x^2 + 49x - 7\)  

(n) \(90x^3 - 90x^2 + 20x\)

(o) \(x^3 - 27\)  

(p) \(5x^3 + 4x^2 + 45x + 36\)

(q) \(27x^2 - 3\)  

(r) \(40x^3 + 5\)
**LESSON #21 - THE ZERO PRODUCT LAW**
**COMMON CORE ALGEBRA II**

One of the most important equation solving technique stems from a fact about the number zero that is not true of any other number:

### THE ZERO PRODUCT LAW

If the product of multiple factors is equal to zero then at least one of the factors must be equal to zero.

The law can immediately be put to use in the first exercise. In this exercise, quadratic equations are given already in factored form.

**Exercise #1:** Solve each of the following equations for all value(s) of \( x \).

(a) \((x+7)(x-3)=0\)

(b) \((2x-5)(x-4)=0\)

(c) \(4(3x+2)(4x-3)=0\)

**Exercise #2:** In Exercise #1(c), why does the factor of 4 have no effect on the solution set of the equation?

The Zero Product Law can be used to solve any quadratic equation that is factorable (not prime). To utilize this technique the problem solver must first set the equation equal to zero and then factor the non-zero side.

**Exercise #3:** Solve each of the following quadratic equations using the Zero Product Law.

(a) \(x^2 + 3x - 14 = -2x + 10\)

(b) \(3x^2 + 12x - 7 = x^2 + 3x - 2\)
**Exercise #4:** Consider the system of equations shown below consisting of a parabola and a line.

\[ y = 3x^2 - 8x + 5 \quad \text{and} \quad y = 4x + 5 \]

(a) Find the intersection points of these curves algebraically.

(b) Using your calculator, sketch a graph of this system on the axes to the right. **Be sure to label the curves with equations, the intersection points, and the window.**

(c) Verify your answers to part (a) by using the INTERSECT command on your calculator.

The Zero Product Law is extremely important in finding the zero’s or **x-intercepts (zeroes)** of a parabola.

**Exercise #5:** A parabola is shown to the right.

(a) Write the coordinates of the two **x-intercepts** of the graph.

(b) Write the equation of the parabola.

**Exercise #6:** *Algebraically* find the set of x-intercepts (zeroes) for each parabola given below.

(a) \( y = 4x^2 - 1 \)  
(b) \( y = 3x^2 + 13x - 10 \)  
(c) \( y = 5x^2 - 10x \)
LESSON #21 - THE ZERO PRODUCT LAW
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Solve each of the following equations for all value(s) of $x$.

   (a) $(x - 2)(x + 5) = 0$
   (b) $(7x - 1)(2x + 5) = 0$
   (c) $(3x - 1)(3x + 1) = 0$

2. Solve each of the following quadratic equations which have already been set equal to zero.

   (a) $x^2 + 10x + 16 = 0$
   (b) $3x^2 + 11x - 4 = 0$
   (c) $12x^2 + 8x = 0$

3. Solve each of the following quadratic equations by first manipulating them so that one side of the equation is set equal to zero.

   (a) $x^2 + 4x - 40 = 10x + 15$
   (b) $4x^2 + 3x - 11 = 3x - 2$

   (c) $6x^2 - 15x + 2 = 2x^2 + 10x - 4$
   (d) $-16t^2 + 76t + 5 = 12t + 5$
APPLICATIONS

4. Consider the system of equations shown below consisting of one linear and one quadratic equation.

\[ y = 4x - 5 \quad \text{and} \quad y = 2x^2 - 5x - 10 \]

(a) Find the intersection points of this system algebraically.

(b) Using your calculator, sketch a graph of this system to the right. Be sure to label the curves with equations, the intersection points, and the window.

(c) Use the INTERSECT command on your calculator to verify the results you found in part (a).

5. Algebraically, find the zeroes (x-intercepts) of each quadratic function given below.

(a) \[ y = x^2 - 81 \]

(b) \[ y = 12x^2 - 18x \]

(c) \[ y = 2x^2 - 6x - 8 \]

REASONING

6. A quadratic function of the form \( y = x^2 + bx + c \).

(a) What are the \( x \)-intercepts of this parabola?

(b) Based on your answer to part (a), write the equation of this quadratic function first in factored form and then in trinomial form.
Lesson #22 - Solving Incomplete Quadratics and Completing the Square

Common Core Algebra II

Quadratics in the form $ax^2 + c = 0$ are known as incomplete. Because these equations lack a linear (b) term they can be solved without the use of factoring and the Zero Product Law.

Exercise #1: Solve each of the following incomplete quadratics for all values of x.

(a) $x^2 - 16 = 0$
(b) $5x^2 - 8 = 12$
(c) $\frac{2}{9}x^2 + 4 = 22$

Unlike those quadratics that we factored and used the Zero Product Rule to solve, incomplete quadratics can have irrational answers as solutions.

Exercise #2: Solve each of the following incomplete quadratics for all values of x. Place all answers in simplest radical form.

(a) $3x^2 - 5 = 19$
(b) $10x^2 + 1 = 6$
(c) $4x^2 + 5 = 8$

Any quadratic equation can be rewritten in a form where the method of Square roots can be used. This process is known as completing the square.

Example: Solve the equation, $x^2 - 6x + 8 = 0$.

1. Move the constant term to the other side of the equation.
2. Find $\frac{b}{2}$ and $\left(\frac{b}{2}\right)^2$
3. Complete the square by adding $\left(\frac{b}{2}\right)^2$ to both sides of the equation.
4. Factor the left side of the equation into a perfect square binomial.
5. Take the square root of both sides. Do not forget the plus or minus.
6. Add or subtract to solve for x.

\[
x^2 - 6x - 8 = 0
\]

1. $x^2 - 6x + \boxed{9} = 8$
2. $\frac{b}{2} = -3$  $\left(\frac{b}{2}\right)^2 = 9$
3. $x^2 - 6x + \boxed{9} = 8 + \boxed{9}$
4. $(x - 3)(x - 3) = 17$
   $(x - 3)^2 = 17$
5. $x - 3 = \pm\sqrt{17}$
6. $x = 3 \pm \sqrt{17}$
You can use this process to solve any quadratic, but it is especially useful when the quadratic cannot be factored.

**Exercise #3:** Solve each of the following prime quadratic equations by first completing the square. Express your answers in simplest radical form.

(a) \( x^2 + 10x + 23 = 0 \)  
(b) \( x^2 + 12x + 18 = 0 \)

**Exercise #4:** For each of the following quadratics, express your answers to the nearest hundredth. Graph the quadratic to verify that you have found the correct answer for the zeroes.

(a) \( x^2 + 2x = 12 \)  
(b) \( x^2 - 14x + 7 = 0 \)

Quadratic equations where \( b \) is even and \( a=1 \) are the easiest to solve by completing the square. When \( b \) is odd, fractions are involved in the process.

**Exercise #5:** Solve each of the following quadratic equations by completing the square.

(a) \( x^2 - 5x + 3 = 0 \)  
(b) \( x^2 - 9x - 12 = 0 \)
You cannot complete the square when $a$ is greater than one. In those cases, divide the entire equation by “$a$” first and then complete the square.

c) $4x^2 - 8x - 20 = 0$  
d) $3x^2 + 9x + 3 = 0$
LESSON #22 - SOLVING INCOMPLETE QUADRATICS AND COMPLETING THE SQUARE
COMMON CORE ALGEBRA II HOMEWORK

1. Solve each of the following incomplete quadratics. Express your answers in simplest radical form when necessary.

a) \(3x^2 = 108\)  
b) \(\frac{1}{2}x^2 \cdot 7 = 25\)  
c) \(500x^2 \cdot 5 = 0\)

d) \(5x^2 = 100\)  
e) \(2x^2 \cdot 20 = 70\)  
f) \(6x^2 + 10 = 12\)

2. Solve each of the following quadratic equations by completing the square. Express your answer in simplest radical form.

a) \(x^2 - 2x - 2 = 0\)  
b) \(x^2 + 6x + 4 = 0\)

c) \(x^2 - 5x + 1 = 0\)  
d) \(2x^2 - 8x + 2 = 0\)
e) \(4x^2 - 20x + 8 = 0\)  \quad f) \(x^2 - 7x + 2 = 0\)

3. Rounded to the nearest hundredth the larger root of \(x^2 - 22x + 108 = 0\) is

(1) 18.21  \quad (3) 6.74

(2) 13.25  \quad (4) 14.61
LESSON #23 - THE QUADRATIC FORMULA
COMMON CORE ALGEBRA II

Simplify each of the following expressions:

\[
\frac{3 \pm \sqrt{11}}{6} = \frac{3 \pm \sqrt{50}}{6} = \frac{3 \pm \sqrt{72}}{6} = \frac{3 \pm \sqrt{25}}{6} =
\]

In the last lesson, you solved the following quadratic equation, \( x^2 - 10x + 23 = 0 \) by completing the square. The solutions were \( x = 5 - \sqrt{2} \) and \( x = 5 + \sqrt{2} \).

Since any quadratic can be rearranged through the process of Completing the Square, a formula can be developed that will solve for the roots of any quadratic equation. This famous formula, known as the Quadratic Formula, is shown below. You worked with this as well in Algebra I.

THE QUADRATIC FORMULA

The solutions to the quadratic equation \( ax^2 + bx + c = 0 \), assuming \( a \neq 0 \), are given by

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Exercise #1: Using the quadratic formula shown above, solve the equation \( x^2 - 10x + 23 = 0 \). You should get the same solution as you did in the last lesson.

How can you tell from your solutions that the quadratic equation, \( x^2 - 10x + 23 = 0 \) is not factorable?

Exercise #2: Which of the following represents the solutions to the equation \( x^2 - 10x + 20 = 0 \)?

\[
(1) \ x = 5 \pm \sqrt{10} \quad (3) \ x = -5 \pm \sqrt{10} \\
(2) \ x = -5 \pm \sqrt{5} \quad (4) \ x = 5 \pm \sqrt{5}
\]
Although the quadratic formula is most helpful when a quadratic expression is **prime** (not factorable), it can be used as a replacement for the Zero Product Law in cases where the quadratic can be factored.

**Exercise #3:** Solve the quadratic equation shown below in two different ways – (a) by factoring and (b) by using the quadratic formula.

(a) $2x^2 + 11x - 6 = 0$ by factoring

(b) $2x^2 + 11x - 6 = 0$ by the quadratic formula

(c) Where will the function, $f(x) = 2x^2 + 11x - 6$ intersect the x-axis?

The quadratic formula is very useful in algebra - it should be committed to memory with practice.

**Exercise #4:** Solve each of the following quadratic equations by using the quadratic formula. Some answers will be purely rational numbers and some will involve irrational numbers. Place all answers in simplest form.

(a) $3x^2 + 5x + 2 = 0$

(b) $x^2 - 8x + 13 = 0$

(c) $2x^2 - 2x - 5 = 0$

(d) $5x^2 + 8x - 4 = 0$
Exercise #5: A shot-put throw can be modeled using the equation \( y = -0.0241x^2 + x + 5.5 \) where \( x \) is the distance traveled (in feet) and \( y \) is the height (also in feet). How long was the throw, to the nearest tenth of a foot?

Let \( x = \)

Let \( y = \)
LESSON #23 - THE QUADRATIC FORMULA
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Solve each of the following quadratic equations using the quadratic formula. Express all answers in simplest form.

(a) $x^2 + 7x - 18 = 0$

(b) $x^2 - 2x - 1 = 0$

(c) $x^2 + 8x + 13 = 0$

(d) $3x^2 - 2x - 3 = 0$

(e) $6x^2 - 7x + 2 = 0$

(f) $5x^2 + 3x - 4 = 0$
2. Which of the following represents all solutions of \( x^2 - 4x - 1 = 0 \)?

- (1) \( 2 \pm \sqrt{5} \)
- (2) \( -2 \pm \sqrt{5} \)
- (3) \( 2 \pm \sqrt{10} \)
- (4) \( -2 \pm \sqrt{12} \)

3. Which of the following is the solution set of the equation \( 4x^2 - 12x - 19 = 0 \)?

- (1) \( \frac{5}{2} \pm \sqrt{3} \)
- (2) \( -\frac{2}{3} \pm \sqrt{2} \)
- (3) \( \frac{3}{2} \pm \sqrt{7} \)
- (4) \( -\frac{7}{3} \pm \sqrt{6} \)

4. Rounded to the nearest hundredth the larger root of \( x^2 - 22x + 108 = 0 \) is

- (1) 18.21
- (2) 13.25
- (3) 6.74
- (4) 14.61

5. Algebraically find the \( x \)-intercepts of the quadratic function whose equation is \( y = x^2 - 4x - 6 \). Express your answers in simplest radical form.

**APPLICATIONS**

6. A missile is fired such that its height above the ground is given by \( h = -9.8t^2 + 38.2t + 6.5 \), where \( t \) represents the number of seconds since the rocket was fired. Using the quadratic formula, determine, to the nearest tenth of a second, when the rocket will hit the ground.

Let \( t = \)

Let \( h = \)
Exercise #1: You have seen that some quadratics are factorable and some are not. You also know that certain methods can always be used to solve a quadratic.

a) What are these methods?

b) Let’s say you used the quadratic formula to solve a quadratic. How can you tell from your answers when you could have also factored the quadratic?

c) Decide if each set of numbers is rational, irrational, or not real.

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<th>i. ( \frac{3 + \sqrt{7}}{2}, \frac{3 - \sqrt{7}}{2} )</th>
<th>ii. ( \frac{3 + \sqrt{25}}{2}, \frac{3 - \sqrt{25}}{2} )</th>
<th>iii. ( \frac{3 + \sqrt{0}}{2}, \frac{3 - \sqrt{0}}{2} )</th>
<th>iv. ( \frac{3 + \sqrt{-9}}{2}, \frac{3 - \sqrt{-9}}{2} )</th>
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d) When do you usually see numbers in the form above?

e) What part of each number dictates what type of number it is?

In the quadratic formula, \( b^2 - 4ac \) is the expression in the radical. It is known as the discriminant because it helps you discriminate (differentiate) between quadratics that can be factored and those that cannot be factored. (It also gives other information that will be covered later.)

Exercise #2: Use the discriminant to quickly determine if each of the following quadratics can be factored. Indicate if the quadratic has solutions that are not real numbers.

(a) \( 2x^2 - 3x + 4 = 0 \)  
(b) \( 3x^2 - 7x + 6 = 0 \)  
(c) \( x^2 - 2 = 5x \)  
(d) \( x^2 - 6x = 9 \)
**Exercise #3:** Consider the quadratic function $f(x) = x^2 - 4x - 36$.

(a) Algebraically determine this function’s $x$-intercepts using the quadratic formula. Express your answers in simplest radical form.

(b) Express the $x$-intercepts of the quadratic to the nearest hundredth.

(c) Using your calculator, sketch a graph of the quadratic on the axes given. Use the INTERSECT command on your calculator to verify your answers from part (b). Label the zeros on the graph.

**Exercise #4:** Which of the following sets represents the $x$-intercepts of $y = 3x^2 - 19x + 6$?

1. $\left\{\frac{1}{2}, \frac{7}{3}\right\}$
2. $\left\{\frac{1}{6} + \frac{\sqrt{17}}{2}, \frac{1}{6} - \frac{\sqrt{17}}{2}\right\}$
3. $\{2 - \sqrt{5}, 2 + \sqrt{5}\}$
4. $\left\{\frac{1}{3}, 6\right\}$
**Exercise #5:** The Crazy Carmel Corn company modeled the percent of popcorn kernels that would pop, \( P \), as a function of the oil temperature, \( T \), in degrees Fahrenheit using the equation

\[
P = -\frac{1}{250} T^2 + 2.8T - 394
\]

The company would like to find the lowest temperature that ensures that 50% of the kernels will pop. Write an equation to model this situation. Solve this equation with the help of the quadratic formula. Round the temperature to the nearest tenth of a degree.

**Exercise #4:** Find the intersection points of the linear-quadratic system shown below algebraically. Then, use your calculator to help produce a sketch of the system. Label the intersection points you found on your graph.

\[
y = 4x^2 - 6x + 2 \quad \text{and} \quad y = 6x - 3
\]
LESSON #24 - MORE WORK WITH QUADRATIC EQUATIONS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Use the discriminant to quickly determine if each of the following quadratics can be factored. If the equation can be factored, solve by factoring. If the equation cannot be factored, choose a different method to solve it.

(a) \( 5x^2 - 6x - 2 = 0 \)  
(b) \( 2x^2 + 11x + 14 = 0 \)

2. Which of the following represents the solutions to \( x^2 - 4x + 12 = 6x - 2 \)?

(1) \( x = 4 \pm \sqrt{7} \)  
(3) \( x = 5 \pm \sqrt{22} \)

(2) \( x = 5 \pm \sqrt{11} \)  
(4) \( x = 4 \pm \sqrt{13} \)

3. The smaller root, to the nearest hundredth, of \( 2x^2 - 3x - 1 = 0 \) is

(1) -0.28  
(3) 1.78

(2) -0.50  
(4) 3.47

4. The \( x \)-intercepts of \( y = 2x^2 + 7x - 30 \) are

(1) \( x = \frac{-7 \pm \sqrt{191}}{2} \)  
(3) \( x = -6 \) and \( \frac{5}{2} \)

(2) \( x = -3 \) and 5  
(4) \( x = -3 \pm \sqrt{131} \)
5. Solve the following equation for all values of $x$. Express your answers in simplest radical form.

$$4x^2 - 4x - 5 = 8x + 6$$

6. Algebraically solve the system of equations shown below.

$$y = 6x^2 + 19x - 15 \text{ and } y = -12x + 15$$

APPLICATIONS

7. The Celsius temperature, $C$, of a chemical reaction increases and then decreases over time according to the formula

$$C(t) = -\frac{1}{2}t^2 + 8t + 93,$$

where $t$ represents the time in minutes. Use the Quadratic Formula to help determine the amount of time, to the nearest tenth of a minute, it takes for the reaction to reach 110 degrees Celsius.
LESSON #25 - IMAGINARY NUMBERS
COMMON CORE ALGEBRA II

Recall that in the Real Number System, it is not possible to take the square root of a negative quantity because whenever a real number is squared it is non-negative. This fact has a ramification for finding the x-intercepts of a parabola, as Exercise #1 will illustrate.

Exercise #1: On the axes below, a sketch of $y = x^2$ is shown. Now, consider the parabola whose equation is given in function notation as $f(x) = x^2 + 1$.

(a) How is the graph of $y = x^2$ shifted to produce the graph of $f(x)$?

(b) Create a quick sketch of $f(x)$ on the axes below.

(c) What can be said about the x-intercepts of the function $y = f(x)$?

(d) Algebraically, show that these intercepts do not exist, in the Real Number System, by solving the incomplete quadratic $x^2 + 1 = 0$.

Since we cannot solve this equation using Real Numbers, we introduce a new number, called $i$, the basis of imaginary numbers. Its definition allows us to now have a result when finding the square root of a negative real number. Its definition is given below.

THE DEFINITION OF THE IMAGINARY NUMBER $i$

$$i = \sqrt{-1}$$

Exercise #2: Simplify each of the following square roots in terms of $i$.

(a) $\sqrt{-9}$

(b) $\sqrt{-100}$

(c) $\sqrt{-32}$

(d) $\sqrt{-18}$
**Exercise #2:** Solve each of the following incomplete quadratics. Place your answers in simplest radical form.

(a) \(5x^2 + 8 = -12\)  
(b) \(\frac{1}{2}x^2 + 20 = 2\)  
(c) \(2x^2 - 10 = -36\)

**Exercise #3:** Which of the following is equivalent to \(5i \cdot 6i\) ?

(1) \(30i\)  
(2) \(11i\)  
(3) \(-30\)  
(4) \(-11\)

Powers of \(i\) display a remarkable pattern that allow us to simplify large powers of \(i\) into one of 4 cases. This pattern is discovered in Exercise #4.

**Exercise #4:** Simplify each of the following powers of \(i\).

\[
i^0 = \quad i^1 = \quad i^2 = \quad i^3 = \\
i^4 = \quad i^5 = \quad i^6 = \quad i^7 =
\]

We see, then, from this pattern that every power of \(i\) is either \(-1, 1, i,\) or \(-i\). And the pattern will repeat.

**Exercise #5:** From the pattern of Exercise #4, simplify each of the following powers of \(i\).

(a) \(i^{38}\)  
(b) \(i^{21}\)  
(c) \(i^{83}\)  
(d) \(i^{40}\)

**Exercise #6:** Which of the following is equivalent to \(5i^{16} + 3i^{23} + i^{26}\) ?

(1) \(8 + 2i\)  
(2) \(4 - 3i\)  
(3) \(5 - 4i\)  
(4) \(2 + 7i\)
LESSON #25 - IMAGINARY NUMBERS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. The imaginary number \( i \) is defined as
   
   (1) \(-1\)    (3) \( \sqrt{-4} \)
   
   (2) \( \sqrt{-1} \)    (4) \( (-1)^2 \)

2. Which of the following is equivalent to \( \sqrt{-128} \)?

   (1) \( 8 \sqrt{2} \)    (3) \( -8 \sqrt{2} \)
   
   (2) \( 8i \)    (4) \( 8i \sqrt{2} \)

3. The sum \( \sqrt{-9} + \sqrt{-16} \) is equal to

   (1) \( 5 \)    (3) \( 7i \)
   
   (2) \( 5i \)    (4) \( 7 \)

4. Which of the following powers of \( i \) is not equal to one?

   (1) \( i^{16} \)    (3) \( i^{32} \)
   
   (2) \( i^{26} \)    (4) \( i^{48} \)

5. Which of the following represents all solutions to the equation \( \frac{1}{3}x^2 + 10 = 7 \)?

   (1) \( x = \pm 3i \)    (3) \( x = \pm i \)
   
   (2) \( x = \pm 5i \)    (4) \( x = \pm 2i \)

6. Solve each of the following incomplete quadratics. Express your answers in simplest radical form.

   (a) \( 2x^2 + 100 = -62 \)
   
   (b) \( \frac{2}{3}x^2 + 20 = 2 \)
7. Which of the following represents the solution set of $\frac{1}{2}x^2 - 12 = -37$?

(1) $\pm 7i$  
(2) $\pm 7i\sqrt{2}$  
(3) $\pm 5i\sqrt{2}$  
(4) $\pm 3i\sqrt{2}$

8. Simplify each of the following powers of $i$ into either $-1, 1, i,$ or $-i$.

(a) $i^2$  
(b) $i^3$  
(c) $i^4$  
(d) $i^{11}$

(e) $i^{41}$  
(f) $i^{30}$  
(g) $i^{25}$  
(h) $i^{36}$

(i) $i^{51}$  
(j) $i^{45}$  
(k) $i^{80}$  
(l) $i^{70}$

9. Which of the following is equivalent to $i^7 + i^8 + i^9 + i^{10}$?

(1) 1  
(2) 2 + i  
(3) 1 − i  
(4) 0

10. When simplified the sum $5i^{18} + 7i^{25} + 2i^{28} + 6i^{43}$ is equal to

(1) $2 − 4i$  
(2) $−3 + i$  
(3) $5 − 7i$  
(4) $8 + i$

11. The product $(6 + 2i)(4 − 3i)$ can be written as

(1) $24 − 6i$  
(2) $18 + 10i$  
(3) $2 + 5i$  
(4) $30 − 10i$
All numbers fall into a very broad category known as complex numbers. Complex numbers can always be thought of as a combination of a real number with an imaginary number and will have the form:

$$a + bi$$ where $a$ and $b$ are real numbers

We say that $a$ is the real part of the number and $bi$ is the imaginary part of the number. These two parts, the real and imaginary, cannot be combined. Like real numbers, complex numbers may be added and subtracted. The key to these operations is that real components can combine with real components and imaginary with imaginary.

**Exercise #1:** Find each of the following sums and differences.

(a) $(-2 + 7i) + (6 + 2i)$  
(b) $(8 + 4i) + (12 - i)$  
(c) $(5 + 3i) - (2 - 7i)$  
(d) $(-3 + 5i) - (-8 + 2i)$

**Exercise #2:** Which of the following represents the sum of $(6 + 2i)$ and $(-8 - 5i)$?

(1) $5i$  
(2) $-2 - 3i$  
(3) $2 + 3i$  
(4) $-5i$

Adding and subtracting complex numbers is straightforward because the process is similar to combining algebraic expressions that have like terms. The complex numbers are closed under addition and subtraction, i.e. when you add or subtract two complex numbers the results is a complex number as well. But, is multiplication closed?

**Exercise #3** Find the following products. Write each of your answers as a complex number in the form $a + bi$.

(a) $(3 + 5i)(7 + 2i)$  
(b) $(-2 + 6i)(3 - 2i)$  
(c) $(4 + i)(-5 - 3i)$
**Exercise #4:** Consider the more general product $(a+bi)(c+di)$ where constants $a$, $b$, $c$ and $d$ are real numbers.

(a) Show that the real component of this product will always be $ac - bd$.

(b) Show that the product of $2+3i$ and $4-6i$ results in a purely real number.

(c) Under what conditions will the product of two complex numbers always be a purely imaginary number? Check by generating a pair of complex numbers that have this type of product.

**Exercise #5:** Determine the result of the calculation below in simplest $a+bi$ form.

$$ (5+2i)(-3+i) + 4i(2+3i) $$

**Exercise #6:** Which of the following products would be a purely real number?

1. $(4+2i)(3-i)$
2. $(-3+i)(-2+4i)$
3. $(5+2i)(5-2i)$
4. $(6+3i)(6+3i)$
LESSON #26 - COMPLEX NUMBERS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Find each of the following sum or difference.
   (a) \((6 + 3i) + (-2 + 9i)\)
   (b) \((-7 + i) - (3 + 5i)\)
   (c) \((10 - 3i) + (6 - 8i)\)

   (d) \((-2 + 7i) - (15 - 6i)\)
   (e) \((15 + 2i) + (5 - 5i)\)
   (f) \((-1 + i) - (-5 - 6i)\)

2. Which of the following is equivalent to \(3(5 + 2i) - 2(3 - 6i)\)?
   (1) \(9 + 18i\)
   (2) \(21 + 8i\)
   (3) \(9 - 6i\)
   (4) \(21 - 2i\)

3. Find each of the following products in simplest \(a + bi\) form.
   (a) \((5 - 2i)(-1 + 7i)\)
   (b) \((3 + 9i)(2 + 4i)\)
   (c) \((-4 - i)(-2 + 6i)\)

4. Complex conjugates are two complex numbers that have the form \(a + bi\) and \(a - bi\). Find the following products of complex conjugates:
   (a) \((5 - 7i)(5 + 7i)\)
   (b) \((10 + i)(10 - i)\)
   (c) \((-3 + 8i)(-3 - 8i)\)

   (d) What's true about the product of two complex conjugates?
5. Show that the product of $a + bi$ and $a - bi$ is the purely real number $a^2 + b^2$.

6. The product of $(-8 + 2i)$ and its conjugate is equal to

(1) $64 + 4i$
(2) $60$
(3) $68$
(4) $60 - 4i$

7. The complex computation $(6 + 2i)(6 - 2i) - (3 - 4i)(3 + 4i)$ can be simplified to

(1) $15$
(2) $39$
(3) $-10$
(4) $-35$

8. Perform the following complex calculation. Express your answer in simplest $a + bi$ form.

$$(8 + 5i)(3 + 2i) - (4 + i)(4 - i)$$

9. Perform the following complex calculation. Express your answer in simplest $a + bi$ form.

$$7(3 - 5i) + (4 - 2i)(-6 + 7i)$$

10. Simplify the following complex expression. Write your answer in simplest $a + bi$ form.

$$\left((5 + 2i)^2 + (2 - i)^2\right)$$
LESSON #27 - SOLVING QUADRATIC EQUATIONS WITH COMPLEX SOLUTIONS
COMMON CORE ALGEBRA II

As we saw in the last unit, the roots or zeroes of any quadratic equation can be found using the quadratic formula:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

Since this formula contains a square root, it is fair to investigate solutions to quadratic equations now when the quantity \( b^2 - 4ac \), known as the discriminant, is negative. Up to this point, we would have concluded that if the discriminant was negative, the quadratic had no (real) solutions. But, now it can have complex solutions.

**Exercise #1:** Use the quadratic formula to find all solutions to the following equation. Express your answers in simplest \( a + bi \) form.

\[ x^2 - 4x + 29 = 0 \]

As long as our solutions can include complex numbers, then any quadratic equation can be solved for two roots.

**Exercise #2:** Solve each of the following quadratic equations. Express your answers in simplest \( a + bi \) form.

(a) \( x^2 - 5x + 30 = 7x - 10 \) 
(b) \( x^2 + 16x + 15 = 10x + 4 \)
There is an interesting connection between the $x$-intercepts (zeroes) of a parabola and complex roots with non-zero imaginary parts. The next exercise illustrates this important concept.

**Exercise #3:** Consider the parabola whose equation is $y = x^2 - 6x + 13$.

(a) Algebraically find the $x$-intercepts of this parabola. Express your answers in simplest $a + bi$ form.

(b) Using your calculator, sketch a graph of the parabola on the axes below. Use the window indicated.

(c) From your answers to (a) and (b), what can be said about parabolas whose zeroes are complex roots with non-zero imaginary parts?

**Exercise #4:** Use the *discriminant* of each of the following quadratics to determine whether it has $x$-intercepts. Then determine if those intercepts could be found by factoring.

(a) $y = x^2 - 3x - 10$  
(b) $y = x^2 + 6x + 10$  
(c) $y = 2x^2 + 3x - 4$

**Exercise #5:** Which of the following quadratic functions, when graphed, would not cross the $x$-axis?

(1) $y = 2x^2 + 5x - 3$  
(2) $y = -x^2 - x + 6$  
(3) $y = 4x^2 - 4x + 5$  
(4) $y = 3x^2 - 13x + 4$
LESSON #27 - SOLVING QUADRATIC EQUATIONS WITH COMPLEX SOLUTIONS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Solve each of the following quadratic equations. Express your solutions in simplest $a + bi$ form.

   (a) $x^2 + 4x + 20 = 12x - 5$
   (b) $x^2 = x - 1$

   (c) $2x^2 - 25x + 27 = -15x - 10$
   (d) $8x^2 + 36x + 24 = 12x + 5$

   (e) $x^2 + 6x + 15 = 8x - 2$
   (f) $4x^2 + 38x + 50 = 10x - 35$
2. Which of the following represents the solution set to the equation $x^2 - 2x + 2 = 0$?

   (1) $x = -1$ or $2$  
   (2) $x = 1 \pm 2i$  
   (3) $x = 2 \pm i$  
   (4) $x = 1 \pm i$

3. The solutions to the equation $x^2 + 6x + 11 = 0$ are

   (1) $x = -3 \pm i\sqrt{2}$  
   (2) $x = -3 \pm 2i\sqrt{2}$  
   (3) $x = -6 \pm i\sqrt{11}$  
   (4) $x = -6 \pm 2i\sqrt{11}$

4. Using the discriminant, $b^2 - 4ac$, determine whether each of the following quadratics has real or imaginary zeroes. Then determine if those intercepts zeroes could be found by factoring.

   (a) $y = 2x^2 - 7x + 6$  
   (b) $y = 3x^2 + 2x + 1$  
   (c) $y = x^2 - 8x + 14$

   (d) $y = 2x^2 - 12x + 26$  
   (e) $y = -2x^2 + 6x - 5$  
   (f) $y = 4x^2 - 4x + 1$

5. Which of the following quadratics, if graphed, would lie entirely above the x-axis? Try to use the discriminant to solve this problem and then graph to check.

   (1) $y = 2x^2 + x - 21$  
   (2) $y = x^2 - x - 6$  
   (3) $y = x^2 - 4x + 7$  
   (4) $y = x^2 - 10x + 16$

**REASONING**

6. For what values of $c$ will the quadratic $y = x^2 + 6x + c$ have no real zeroes? Set up and solve an inequality for this problem.