**LESSON #42 - INVERSES OF FUNCTIONS AND FUNCTION NOTATION – PART 2**

**COMMON CORE ALGEBRA II**

You will recall from unit 1 that in order to find the inverse of a function, you must switch x and y and solve for y. Also, for a function, f(x), the inverse function is written as \( f^{-1}(x) \).

**Exercise #2:** Find \( f^{-1}(x) \).

(a) \( f(x) = 3x + 2 \)

(b) \( f(x) = \frac{1}{2}x + 5 \)

The inverses of polynomial and rational functions can also be found.

**Exercise #3:** Find formulas for the inverse of each of the following simple rational or polynomial functions.

(a) \( g(x) = \frac{x}{x-2} \)

(b) \( h(x) = \frac{x+3}{2x} \)

(c) \( y = 2(x-2)^3 \)

(d) \( f(x) = \frac{4+\sqrt[3]{4x}}{2} \)

(e) \( y = \frac{x+4}{x-2} \)

(f) \( f(x) = (x+4)^5 - 6 \)
In the next lesson, we will be working on transforming functions. For this lesson, we will practice writing new functions that are related to these transformations.

**Exercise #4:** The following table includes two parent functions and a quadratic function. Complete each of the following function changes. You do not need to simplify the resulting function. **Do not do the last column.**

Each function change is written as if the function were \( g(x) \). Make the same change to \( m(x) \) and \( h(x) \).

<table>
<thead>
<tr>
<th>Function Change</th>
<th>A. Cubic Parent Function ( g(x) = x^3 )</th>
<th>B. Square Root Parent Function ( m(x) = \sqrt{x} )</th>
<th>C. Quadratic Function ( h(x) = x^2 - 4x + 3 )</th>
<th>Transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(-x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-f(x))</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x+2) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x)-2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x-2)+5 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 3f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(2x) )</td>
<td></td>
<td></td>
<td>****</td>
<td></td>
</tr>
<tr>
<td>( \frac{1}{2}f(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f\left(\frac{1}{3}x\right) )</td>
<td></td>
<td>****</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 2f(x+3) )</td>
<td></td>
<td>****</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(-x)-5 )</td>
<td></td>
<td>****</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
LESSON #42 - INVERSES OF FUNCTIONS AND FUNCTION NOTATION – PART 2
COMMON CORE ALGEBRA II HOMEWORK

1. Find formulas for the inverse of each of the following functions.

(a) \( y = \frac{5x}{x - 2} \)  
(b) \( y = \frac{3x + 2}{x + 4} \)

(c) \( f(x) = 2x^3 + 3 \)  
(d) \( g(x) = (x + 1)^3 + 2 \)

(e) \( h(x) = \sqrt[3]{x} \cdot 3 \)  
(f) \( g(x) = \frac{1}{x} - 1 \)

(g) \( y = 2(x + 4)^5 - 5 \)  
(h) \( y = \frac{4x - 2}{x + 3} \)
2. Complete the following table. Column A and B are parent functions, and Column C is a cubic function. 

NOTE: Each function change is written as if the function were \( f(x) \). Make the change same change to \( j(x) \) and \( g(x) \). You do not need to simplify.

| Function Change | A. Absolute Value Parent Function \( f(x) = |x| \) | B. Quartic Parent Function \( j(x) = x^4 \) | C. Cubic Function \( g(x) = x^3 - 4x^2 + 3x - 6 \) |
|-----------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| \( f(-x) \)     |                                               |                                               |                                               |
| \(-f(x)\)       |                                               |                                               |                                               |
| \( f(x-4) \)    |                                               |                                               |                                               |
| \( f(x)+1 \)    |                                               |                                               |                                               |
| \( f(x+3)-2 \)  |                                               |                                               |                                               |
| \( 2f(x) \)     |                                               |                                               |                                               |
| \( f(3x) \)     |                                               |                                               |                                               |
| \( \frac{1}{3} f(x) \) |                                           |                                               |                                               |
| \( f\left(\frac{1}{2}x\right)\) |                                         |                                               |                                               |
| \( 4f(x-1) \)   |                                               |                                               |                                               |
| \( -f(x)+6 \)   |                                               |                                               |                                               |
LESSON #43 – FUNCTION TRANSFORMATIONS
COMMON CORE ALGEBRA II

In the previous lesson, we worked with function changes such as \( f(-x) \) and \( f(x+3) \). Each of these changes causes a predictable change in the graph of the function known as a transformation. For example, making the \( x \) negative, \( f(-x) \), will always cause the graph to change in the same way. The easiest function to see all of the transformations is the square root parent function.

**Exercise #1:** Go back to lesson #72, graph the square root parent function as \( Y_1 \) and each of the changes (transformations) as \( Y_2 \). Determine the transformation that occurred for each problem, and write it in the last column.

**Exercise #2:** Summarize your findings in the table below.

<table>
<thead>
<tr>
<th>Function Notation</th>
<th>Transformation</th>
<th>Groups of Transformations</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(-x) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-f(x))</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x+a) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x-a) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x)+a )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(x)-a )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( af(x) \text{ where } a &gt; 1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( af(x) \text{ where } 0 &lt; a &lt; 1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(ax) \text{ where } a &gt; 1 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( f(ax) \text{ where } 0 &lt; a &lt; 1 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: We will be working with horizontal stretches and compressions more extensively in the next unit.
**Exercise #3:** What is the equation of the absolute value parent function, $a(x) = |x|$, after each of the following transformations.

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Shift Right 3</td>
<td>$a(x) = x + 3$</td>
</tr>
<tr>
<td>b. Reflection in the x-axis.</td>
<td>$a(x) = -x$</td>
</tr>
<tr>
<td>c. Shift Down 2</td>
<td>$a(x) = x - 2$</td>
</tr>
<tr>
<td>d. Vertical Stretch of 3.</td>
<td>$a(x) = 3x$</td>
</tr>
<tr>
<td>e. Reflection in the y-axis</td>
<td>$a(x) = -</td>
</tr>
<tr>
<td>f. Shift left 3, up 4.</td>
<td>$a(x) = -x + 4$</td>
</tr>
<tr>
<td>g. Vertical Compression of $\frac{1}{2}$.</td>
<td>$a(x) = \frac{1}{2}x$</td>
</tr>
</tbody>
</table>

**Exercise #4:** The function $f(x)$ is shown on the grid below. A second function, $g(x)$, is defined by $g(x) = f(x-3) + 1$.

(a) Identify how the graph of $f$ has been transformed to produce the graph of $g$ and sketch it on the grid.

(b) A third function, $h$, is defined by $h(x) = 2f(x)$. Identify how the graph of $f$ has been transformed to produce the graph of $h$ and sketch it on the grid. (Note: The maximum of $h$ will be off the grid).

**Exercise #5:** If the parabola $y = x^2$ were shifted 6 units left and 2 units down, its resulting equation would be which of the following? Verify by graphing your answer and seeing if its turning point is at (-6, -2).

1. $y = (x + 6)^2 + 2$
2. $y = (x + 6)^2 - 2$
3. $y = (x - 6)^2 + 2$
4. $y = (x - 6)^2 - 2$
**Exercise #6:** The graph of a function \( f(x) \) is shown below on two grids. Sketch (a) the graph of \(-f(x)\) and (b) the graph of \( f(-x)\).

![Graphs of f(x), -f(x), and f(-x)](image)

**Exercise #7:** Determine an equation for the linear function \( g(x) = 5x - 7 \) both after a reflection in the \( x\)-axis and \( y\)-axis. Label your equations.

**Exercise #8:** If the point \((-3, 12)\) lies on the graph of the function \( y = f(x) \), which of the following points must lie on the graph of \( y = 3f(x) \)?

1. \((-9, 36)\)  
2. \((-3, 36)\)  
3. \((-3, 4)\)  
4. \((-9, 12)\)

**Exercise #9:** If \( f(x) = -2x^2 + 5x - 3 \) and \( g(x) \) is the reflection of \( f(x) \) across the \( y\)-axis, then an equation of \( g \) is which of the following?

1. \( g(x) = -2x^2 - 5x - 3 \)  
2. \( g(x) = -2x^2 + 5x + 3 \)  
3. \( g(x) = 2x^2 + 5x - 3 \)  
4. \( g(x) = 2x^2 + 5x + 3 \)


**Lesson #43 - Function Transformations**

**Common Core Algebra II Homework**

1. What is the equation of the quintic parent function, \( f(x) = x^5 \), after each of the following transformations? You do not need to simplify.

<table>
<thead>
<tr>
<th>Transformation</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Shift left 4</td>
<td>( f(x) = x + 4 )</td>
</tr>
<tr>
<td>b. Reflection in the y-axis</td>
<td>( f(x) = -x^5 )</td>
</tr>
<tr>
<td>c. Shift Up 1</td>
<td>( f(x) = x^5 + 1 )</td>
</tr>
<tr>
<td>d. Vertical Compression of ( \frac{1}{4} )</td>
<td>( f(x) = \frac{1}{4}x^5 )</td>
</tr>
<tr>
<td>e. Reflection in the x-axis</td>
<td>( f(x) = -(x^5) )</td>
</tr>
<tr>
<td>f. Shift right 5, down 2</td>
<td>( f(x) = x^5 - 5 )</td>
</tr>
<tr>
<td>g. Vertical Stretch of 3</td>
<td>( f(x) = 3x^5 )</td>
</tr>
</tbody>
</table>

2. Which of the following represents the turning point of \( f(x) = (x - 8)^2 - 4 \)?

   (1) (8, 4)          (3) (8, 4)
   (2) (8, 4)          (4) (8, 4)

3. Consider the quadratic function \( f(x) = x^2 - 4x - 5 \). The quadratic functions \( g \) and \( h \) are defined by the formula, \( g(x) = 2f(x) \) and \( h(x) = \frac{1}{2}f(x) \). Determine formulas for both \( g \) and \( h \) in standard form.

4. Which of the following equations would represent the graph of the parabola \( y = 3x^2 - 4x - 1 \) after a reflection in the \( x \)-axis?

   (1) \( y = -3x^2 + 4x - 1 \)  (3) \( y = 3x^2 + 4x - 1 \)
   (2) \( y = -3x^2 + 4x - 1 \)  (4) \( y = -3x^2 + 4x + 1 \)
5. If the point \((-3, -5)\) lies on the graph of a function \(h(x)\) then which of the following points \(must\) lie on the graph of the function \(-h(x)\)?

(1) (3, 5)  
(2) \((-3, 5)\)

(3) \((-5, -3)\)

(4) (3, -5)

6. If the point \((-6, 10)\) lies on the graph of \(y = f(x)\) then which of the following points \(must\) lie on the graph of \(y = \frac{1}{2} f(x)\)?

(1) (-3, 5)

(2) \((-3, 10)\)

(3) (-6, 5)

(4) (-12, 20)

7. If the quadratic function \(f(x)\) has a turning point at \((-3, 7)\) then where does the quadratic function \(g\) defined by \(g(x) = f(x + 4) + 5\) have a turning point?

(1) \((-7, 12)\)

(2) \((1, 12)\)

(3) \((-4, 5)\)

(4) \((4, 5)\)

8. The graph of \(f(x) = x^2 + 4x\) is show below on two separate grids. Give an equation and sketch a graph for the functions (a) \(f(-x)\) and (b) \(-f(x)\).
9. Given the function $f(x)$ shown graphed on the grid, create a graph for each of the following functions and label on the grid.

(a) $g(x) = f(x) + 2$

(b) $h(x) = f(x - 3)$

(c) $k(x) = \frac{1}{2}f(x)$

10. The graph of the function $f(x)$ is shown on the grid below. The function $g$ is defined by the formula $g(x) = f(x + 3) - 1$.

(a) Graph and label $g$ on the axes.

(b) What is the smallest solution to the equation $f(x) = g(x)$?

(c) If $h(x) = g(x) - 3$, explain why the equation $h(x) = f(x)$ has no solutions.
LESSON #44 – MULTIPLE TRANSFORMATIONS
COMMON CORE ALGEBRA II

The following activity will help you start to think about multiple transformations.

Exercise #1: Identify the parent function for each transformation.
Draw arrows to each transformation that has happened to the parent function and identify them.

a. \( g(x) = 3\sqrt{-x} - 7 \)

b. \( y = 3x - 7 \)

c. \( h(x) = -|x - 2| \)

d. \( r(x) = 5(x - 3)^2 + 9 \)

Exercise #2: Consider the function, \( g(x) = \sqrt{x + 5} + 3 \)

(a) Graph the function \( y = g(x) \) on the grid shown.

(b) Describe the transformations that have occurred to the graph of \( y = \sqrt{x} \) to produce the graph of \( y = g(x) \).
Specify the transformations and the order.

Exercise #3: How would the graph of the function \( h(x) \) compare to the graph of \( f(x) \) if \( h \) is defined by the formula \( h(x) = f(-x) \)?
**Exercise #4:** Which of the following equations represents the graph shown below?

(1) \( y = (x+3)^2 + 4 \)
(2) \( y = -(x+3)^2 + 4 \)
(3) \( y = -(x-3)^2 + 4 \)
(4) \( y = -(x-3)^2 - 4 \)

**Exercise #5:** Consider the function \( g(x) = -x^2 + 4 \). What two transformations have occurred to the graph of \( y = x^2 \) to produce the graph of \( g \)? Specify the transformations and the order in which they occurred. Note that there exists more than one correct answer. Graph on your calculator to verify.

**Exercise #6:** If the quadratic function \( g(x) \) has a \( y \)-intercept of 12, which of the following is true about the function \( h(x) = 3g(x) - 5 \)?

(1) It has a \( y \)-intercept of -5.
(2) It has a \( y \)-intercept of 21.
(3) It has a \( y \)-intercept of -15.
(4) It has a \( y \)-intercept of 31.

**Exercise #7:** The graph of \( y = h(x) \) is shown below. The function \( f(x) \) is defined by \( f(x) = -\frac{1}{2}h(x) + 3 \).

(a) What three transformations have occurred to the graph of \( h \) to produce the graph of \( f \)? Specify the transformations and the order they occurred in.

(b) Graph and label the function \( f(x) \) on the grid below that contains \( h(x) \).
Exercise #8: The function $h(x)$ has a range given by the interval $[2, 10]$. The function $f(x)$ is defined by $f(x) = \frac{3}{2} h(x) + 8$. Which of the following gives the range of $f(x)$?

(1) [11, 23]  
(2) [8, 12]  
(3) [15, 27]  
(4) [6, 32]
1. For each of the following functions, identify the parent function. In the last column, identify the transformations that occurred on the parent function and their order.

<table>
<thead>
<tr>
<th>New Function</th>
<th>Equation of Parent Function</th>
<th>Transformation(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = x + 6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = 5x^2 - 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = -x^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = \sqrt{x + 7}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = (3x)^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = (x - 4)^2 + 6$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = 3\sqrt{x}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) =</td>
<td>x - 4</td>
<td>- 8$</td>
</tr>
<tr>
<td>$f(x) = \frac{1}{2}x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = \frac{1}{3}\sqrt{x + 2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = 3(x - 1)^2 + 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = -2(x + 9)^3 - 4$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. If the function $h(x)$ is defined as vertical stretch by a factor of 2 followed by a reflection in the $x$-axis of the function $f(x)$ then $h(x) =$

(1) $2f(-x)$  
(2) $\frac{1}{2} f(x)$  
(3) $-\frac{1}{2} f(x)$  
(4) $-2f(x)$

3. If the graph of $y = x^2$ is compressed by a factor of $\frac{1}{3}$ in the $y$-direction and then shifted 4 units down, the resulting graph would have an equation of

(1) $y = \frac{1}{3} x^2 - 4$  
(2) $y = -3x^2 - 4$  
(3) $y = -4x^2 - 3$  
(4) $y = -\frac{1}{3} x^2 + 4$

4. The quadratic function $f(x)$ has a turning point at $(-3, 6)$. The quadratic $y = \frac{2}{3} f(x) + 3$ would have a turning point of

(1) $(-2, 9)$  
(2) $(1, 7)$  
(3) $(-3, 7)$  
(4) $(-1, 9)$

5. The graph of $y = f(x)$ is shown below. Consider the function $y = g(x)$ defined by $g(x) = 2f(x) - 3$.

(a) What two transformations have occurred to the graph of $f$ in order to produce the graph of $g$? Specify both the transformations and their order.

(b) Graph and label $y = g(x)$
Recall that functions are simply rules that convert inputs into outputs. These rules then get placed into various categories, such as linear functions, exponential functions, quadratic functions, etcetera, based on shared characteristics. In this lesson you will learn another way to classify some functions that have useful symmetries.

**EVEN AND ODD FUNCTIONS**

A function is known as **even** if \( f(x) = f(-x) \) for every value of \( x \) in the domain of \( f(x) \).

A function is known as **odd** if \( f(-x) = -f(x) \) for every value of \( x \) in the domain of \( f(x) \).

The terms “even function” and “odd function” come from the properties of power functions that have an even exponent or an odd exponent, respectively.

**Exercise #1:** Draw the following sketches. Identify the type of symmetry for the graph and explain how this makes sense based on the definition above.

(a) Draw a basic sketch of an even power function in the form, \( y = ax^b \) where \( a \) and \( b \) are positive.

(b) Draw a basic sketch of an odd power function in the form, \( y = ax^b \) where \( a \) and \( b \) are positive.

2. In the table below, functions \( f(x) \) and \( g(x) \) are described by their equations, and functions \( h(x) \) and \( k(x) \) are shown by their graphs. Circle whether each function is odd, even or neither.

<table>
<thead>
<tr>
<th>( f(x) = x + 4 )</th>
<th>( g(x) = 3x^2 )</th>
<th>( h(x) )</th>
<th>( k(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Odd, Even, Neither</td>
<td>Odd, Even, Neither</td>
<td>Odd, Even, Neither</td>
<td>Odd, Even, Neither</td>
</tr>
</tbody>
</table>
**Exercise #3:** Consider the partial graph of the function \( f(x) \) shown twice below. Sketch the other half of the function if in (a) \( f(x) \) is **even** and in (b) \( f(x) \) is **odd**. The three coordinate pairs are listed to help you plot.

(a) even

(b) odd

\[(0, 0), (3, 5), (7, -2)\]

**Exercise #4:**

1. Determine whether the following functions are odd, even, or neither. Support algebraically.

   a) \( y = x^2 + 1 \)

   b) \( f(x) = (x + 1)^3 \)

   c) \( y = x^5 - 2x^3 + x \)

**Exercise #5:** Let’s investigate \( g(x) = x^3 - 4x \).

   a) Use your calculator's table option to fill in the following table. What type of function is this? Explain.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( g(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

   (b) Sketch a graph of \( g(x) \) using your calculator and the window indicated.
Determine if the functions on the left side are Even, Odd, or neither. Place a check in the appropriate column. At the bottom of the sheet, use the total number of checks in each column to see if the stated equation is true. Check your work if the expression does not equal 22. Use a table, graph, or algebraic work.

<table>
<thead>
<tr>
<th>Function</th>
<th>Even</th>
<th>Odd</th>
<th>Neither</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = 5x^2 - 4x + 3$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g(x) = 3x^3 - 4x$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h(x) = -2x^4 + 3x^2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = -2x^5 + 3x^3 - 5x^2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n(x) = \sqrt{x^2 + 9}$</td>
<td></td>
<td></td>
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<tr>
<td>$s(x) = 3x^3$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g(x) = \sqrt{1 - x^2}$</td>
<td></td>
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</tr>
<tr>
<td>$h(x) = x^6 + 2x^4 - 3x^2 - 4$</td>
<td></td>
<td></td>
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<tr>
<td>$m(x) = (x - 2)^2$</td>
<td></td>
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</tr>
<tr>
<td>$p(x) = 2x\sqrt{x^2 + 4}$</td>
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</tr>
</tbody>
</table>

Expression Check: $4(\text{# of Evens}) + 3(\text{# of Odds}) - (\text{# of Neither}) = 22$

$4( ) + 3( ) - ( ) = \underline{\hspace{2cm}}$

COMPLETE THIS PAGE AS A PART OF YOUR HOMEWORK.
LEsson #45 - Even and Odd Functions
Common Core Algebra II Homework

Fluency

1. Complete the previous page.

2. Given the partially filled out table below for $f(x)$, fill out the rest of it based on the function type.

   (a) Even
   
   $\begin{array}{c|cccccc}
   x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
   \hline
   y & 5 & -7 & 4 & -4 & & & \\
   \end{array}$

   (b) Odd
   
   $\begin{array}{c|cccccc}
   x & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
   \hline
   y & 5 & -7 & 0 & -4 & & & \\
   \end{array}$

3. Half of the graph of $f(x)$ is shown below. Sketch the other half based on the function type.

   (a) Even
   
   ![Graph of Even Function](image)

   (b) Odd
   
   ![Graph of Odd Function](image)

4. The graphs of four well known function are shown. Circle whether each is odd, even or neither.

   ![Graphs of Functions](image)

   $y = |x|$  Odd, Even, Neither
   $y = \sqrt{x + 4}$  Odd, Even, Neither
   $y = [x]$  Odd, Even, Neither
   $y = e^{-x}$  Odd, Even, Neither
5. If \( f(x) \) is an even function and \( f(3) = 5 \) then what is the value of \( 4f(3) + 2f(-3) \)?

\[
\begin{array}{cc}
(1) \ 30 & (3) \ 10 \\
(2) \ 60 & (4) \ 6 \\
\end{array}
\]

6. Which of the following functions is even? Show algebraically how you arrived at your choice.

\[
\begin{array}{cc}
(1) \ y = x^2 - 4x & (3) \ y = 9 - x^2 \\
(2) \ y = |x - 6| & (4) \ y = 4^x \\
\end{array}
\]

7. The function \( f(x) = \frac{4x^2 + 2}{x} \) is either even or odd. Determine which by exploring the function using tables on your calculator. Copy the table. Show algebraically that the choice is correct.

**Reasoning**

8. Even functions have symmetry across the y-axis. Odd functions have symmetry across the origin. Can a function have symmetry across the x-axis? Why or why not?
Lesson #46 - Circles and Systems

Common Core Algebra II

We will start this lesson by reviewing part of what you learned about circles in common core geometry.

**The Equation of a Circle**

A circle whose center is at \((h, k)\) and whose radius is \(r\) is given by: 
\[(x - h)^2 + (y - k)^2 = r^2\]

**Exercise #1:** Which of the following equations would have a center of \((-3, 6)\) and a radius of 3?

1. \((x - 3)^2 + (y + 6)^2 = 9\)
2. \((x + 3)^2 + (y - 6)^2 = 9\)
3. \((x - 3)^2 + (y - 6)^2 = 3\)
4. \((x + 3)^2 + (y + 6)^2 = 3\)

**Exercise #2:** For each of the following equations of circles, determine both the circle’s center and its radius. If its radius is not an integer, express it in simplest radical form and rounded to the nearest tenth.

(a) \((x - 2)^2 + (y - 7)^2 = 100\)
(b) \((x - 5)^2 + (y + 8)^2 = 4\)
(c) \(x^2 + y^2 = 121\)

(d) \((x + 1)^2 + (y + 2)^2 = 1\)
(e) \(x^2 + (y - 3)^2 = 49\)
(f) \((x + 6)^2 + (y - 5)^2 = 18\)

(g) \(x^2 + y^2 = 64\)
(h) \((x - 4)^2 + (y - 2)^2 = 20\)
(i) \(x^2 + y^2 = 57\)

**Exercise #3:** Based on your answers, graph circles d, g, and h on the grid below. Label each circle.
In Unit #2 you learned how to solve systems of equations involving a quadratic equation and a linear equation. Now we will work with systems that involve a linear equation and a circle. These can be solved both algebraically and graphically.

**Exercise #4**: Solve the following system of equations both algebraically and graphically.

\[ y - 1 = x \]
\[ x^2 + y^2 = 25 \]

**Exercise #5**: Solve the following system of equations both algebraically and graphically:

\[ y + x = 7 \]
\[ (x - 2)^2 + (y - 2)^2 = 9 \]
**Exercise #6:** Solve the following system of equations both algebraically and graphically:

\[
\begin{align*}
y &= x + 5 \\
x^2 + y^2 &= 97
\end{align*}
\]

<table>
<thead>
<tr>
<th>Algebraically:</th>
<th>Graphically:</th>
</tr>
</thead>
</table>

*Note:* It can be difficult to find the exact points of intersection if the circle is not graphed perfectly. An algebraic method is recommended.
LESSON #46 - CIRCLES AND SYSTEMS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Each of the following is an equation of a circle. State the circle’s center and radius. If its radius is not an integer, express it in simplest radical form and rounded to the nearest tenth.

(a) \( x^2 + y^2 = 50 \)  
(b) \( (x - 3)^2 + (x + 7)^2 = 36 \)  
(c) \( (x + 5)^2 + (y + 1)^2 = 64 \)

(d) \( (x - 6)^2 + (y + 6)^2 = 20 \)  
(e) \( x^2 + y^2 = 1 \)  
(f) \( (x - 3)^2 + y^2 = 200 \)

2. Which of the following is true about a circle whose equation is \( (x+5)^2 + (y-3)^2 = 36 \) ?

(1) It has a center of \((5, -3)\) and an area of \(12\pi\).
(2) It has a center of \((-5, 3)\) and a diameter of 6.
(3) It has a center of \((-5, 3)\) and an area of \(36\pi\).
(4) It has a center of \((5, -3)\) and a circumference of \(12\pi\).

3. Which of the following represents the equation of the circle shown graphed below?

(1) \( (x - 2)^2 + (y + 3)^2 = 16 \)
(2) \( (x + 2)^2 + (y - 3)^2 = 4 \)
(3) \( (x - 2)^2 + (y + 3)^2 = 4 \)
(4) \( (x + 2)^2 + (y - 3)^2 = 16 \)
7. Solve the following system of equations both algebraically and graphically. 

\[
\begin{align*}
    y + x &= 10 \\
    x^2 + y^2 &= 58
\end{align*}
\]

<table>
<thead>
<tr>
<th>Algebraically:</th>
<th>Graphically:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image" alt="Graph" /></td>
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</table>

8. Find the intersection of the circle \((x+3)^2 + (y+4)^2 = 29\) and \(y = x - 4\) both algebraically and graphically.

<table>
<thead>
<tr>
<th>Algebraically:</th>
<th>Graphically:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image" alt="Graph" /></td>
</tr>
</tbody>
</table>
**Exercise #1:** Solve each of the systems of equations by elimination.

(a) \[3x + 2y = -9\]
\[2x + y = -7\]

(b) \[4x - 3y = 25\]
\[-3x + 8y = 10\]

You should be very familiar with solving two-by-two systems of linear equations (two equations and two unknowns). These linear systems serve as the basis for a field of math known as **Linear Algebra**.

**Exercise #2:** Consider the three-by-three system of linear equations shown below. Each equation is numbered in this first exercise to help keep track of our manipulations.

\[(1) \quad 2x + y + z = 15\]
\[(2) \quad 6x - 3y - z = 35\]
\[(3) \quad -4x + 4y - z = -14\]

(a) The **addition property of equality** allows us to add two equations together to produce a third valid equation. Create a system by adding equations (1) and (2) and (1) and (3). Why is this an effective strategy in this case?

(b) Use this new two-by-two system to solve the three-by-three.
Just as with two by two systems, sometimes three-by-three systems need to be manipulated by the multiplication property of equality before we can eliminate any variables.

**Exercise #3:** Consider the system of equations shown below. Answer the following questions based on the system.

\[
\begin{align*}
4x + y - 3z &= -6 \\
-2x + 4y + 2z &= 38 \\
5x - y - 7z &= -19
\end{align*}
\]

(a) Which variable will be easiest to eliminate? Why? Use the multiplicative property of equality and elimination to reduce this system to a two-by-two system.

(b) Solve the two-by-two system from (a) and find the final solution to the three-by-three system.
You can easily check your solution to any system of equations by storing your answers for each of the variables and making sure they make each of the equations true. This is especially useful if you get a multiple choice question on this topic. Use the storing method to solve the problem below.

**Exercise #4:** Which ordered triple represents the solution to the system of equations?

\[
2x + y - 3z = 11 \\
-x + 2y + 4z = -3 \\
x - 5y + 2z = -18
\]

a. (-1,1,-4)  
b. (1,3,-2)  
c. (3,8,1)  
d.(2,-3,0)

**Exercise #5:** Solve the system of equations shown below. Show each step in your solution process.

\[
4x - 2y + 3z = 23 \\
x + 5y - 3z = -37 \\
-2x + y + 4z = 27
\]

These are challenging problems only because they are long. Be careful and you will be able to solve each one of these more complex systems.
LESSON #47 - SYSTEMS OF LINEAR EQUATIONS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. Solve the following systems of equations algebraically.

(a) \[
\begin{align*}
2x - 4y &= 8 \\
-3x + y &= 3
\end{align*}
\]

(b) \[
\begin{align*}
2x - 3y &= -2 \\
4x + y &= 24
\end{align*}
\]

2. Show that \(x = 10, y = 4,\) and \(z = 7\) is a solution to the system below \textit{without} solving the system formally.

\[
\begin{align*}
x + 2y + z &= 25 \\
4x - y - 5z &= 1 \\
-2x - y + 8z &= 32
\end{align*}
\]

3. Solve the following system of equations. Carefully show how you arrived at your answers.

\[
\begin{align*}
4x + 2y - z &= 21 \\
-x - 2y + 2z &= 13 \\
3x - 2y + 5z &= 70
\end{align*}
\]
4. Algebraically solve the following system of equations. There are two variables that can be readily eliminated, but your answers will be the same no matter which you eliminate first.

\[
\begin{align*}
2x + 5y - z &= -35 \\
x - 3y + 4z &= 31 \\
-3x + 2y + 2z &= -23 \\
\end{align*}
\]

5. Algebraically solve the following system of equations. This system will take more manipulation because there are no variables with coefficients equal to 1.

\[
\begin{align*}
2x + 3y - 2z &= 33 \\
4x + 5y + 3z &= 54 \\
-6x - 2y - 8z &= -50 \\
\end{align*}
\]
**LESSON #48 – FOCUS AND DIRECTRIX OF A PARABOLA – PART 1**

**COMMON CORE ALGEBRA II**

Materials: 1 sheet patty paper, 1 pencil, 4 colored pencils, ruler

1. Use ruler to draw a straight line with one to two ruler-widths from the bottom of the patty paper
2. Fold paper in half, folding line upon itself making a crease, mark a point above the line on this crease
3. Make several creases in which the line coincides with the point
4. Using a one of the colored pencils outline the shape that the creases form
5. Draw a sketch of the line, the point, and the shape formed in the textbox below.

**DEFINITION OF A PARABOLA**

A parabola is a special curve shaped like an arch.

Any point on a parabola is at an equal distance from…..

* a fixed straight line, ______________, and

*a fixed point, the ______________

Label each of the above in the textbox with a second color.

The **axis of symmetry** is the line that divides a parabola into two parts that are mirror images.

The **vertex** of a parabola is the point at which the parabola intersects the axis of symmetry. On the coordinate plane, the coordinates of the vertex are represented by the general point (h,k).

The vertex is also the **directly between the focus and directrix**.

Draw the axis of symmetry and the vertex in the textbox above with a third color and label them.

The directed **distance** from the vertex to the focus is represented by the variable, p. This new variable p is one you'll need to be able to work with when writing equations of parabolas.

Label p in the textbox with the fourth color.
GEOMETRIC DEFINITION OF A PARABOLA

A parabola is the set of all points \((x,y)\) in a plane that are equidistant from a fixed line (directrix) and a fixed point (focus) not on the line.

The standard form of the equation of a parabola with a vertical axis of symmetry when the vertex \((h,k)\) and the \(p\) value are known is:

\[
y = \frac{1}{4p} (x - h)^2 + k
\]

When the axis of symmetry is a horizontal axis, the formula is:

\[
x = \frac{1}{4p} (y - k)^2 + h
\]

| If the lead coefficient is positive: the parabola opens up. | If the lead coefficient is positive: the parabola opens right. |
| If the lead coefficient is negative: the parabola opens down. | If the lead coefficient is negative: the parabola will open left. |

The important difference in the two equations is which variable is squared: for regular (vertical) parabolas, \(x\) is squared; for sideways (horizontal) parabolas, \(y\) is squared. Note: the values of \(h\) and \(k\) are switched as well so that the y-value of the vertex is with the y-variable.

For the first day, we will be focusing on parabolas with vertical directrixes

**Exercise #1**: For each of the following equations of a parabola (a) state whether it opens up or down and identify the (b) vertex, (c) \(p\)-value, (d) focus, and (e) the equation of the directrix.

1. \[y = \frac{1}{12} (x - 3)^2 + 4\]
   - (a) Opens up or down
   - (b) Vertex
   - (c) \(p\)-value
   - (d) Focus
   - (e) Equation of directrix
2. \( y - 6 = -\frac{1}{4} x^2 \)
(a) Opens up or down
(b) Vertex
(c) \( p \)-value
(d) Focus
(e) Equation of directrix

3. \( 8(y - 8) = (x + 7)^2 \)
(a) Opens up or down
(b) Vertex
(c) \( p \)-value
(d) Focus
(e) Equation of directrix
4. \((x - 3)^2 = y + 2\)

(a) Opens up or down

(b) Vertex

(c) p-value

(d) Focus

(e) Equation of directrix

**Exercise #2**: Write the equation of a parabola with each of the following qualities. It may be helpful to draw a sketch of the parabola first.

a) Write the equation of a parabola whose vertex is \((2,0)\) and directrix is \(y = 6\).

b) Write the equation of a parabola whose directrix is \(y = 2\) and focus is \((-1,8)\).
For each of the following equations of a parabola (a) state whether it opens up or down and identify the (b) vertex, (c) p-value, (d) focus, and (e) the equation of the directrix.

1. \( y = \frac{1}{8} (x + 3)^2 - 1 \)
   (a) Opens up or down
   (b) Vertex
   (c) p-value
   (d) Focus
   (e) Equation of directrix

2. \( y - 4 = -\frac{1}{20} (x + 4)^2 \)
   (a) Opens up or down
   (b) Vertex
   (c) p-value
   (d) Focus
   (e) Equation of directrix

3. \( 12(y + 6) = (x + 4)^2 \)
   (a) Opens up or down
   (b) Vertex
   (c) p-value
   (d) Focus
   (e) Equation of directrix
4. \(-2(x + 1)^2 = y - 4\)
   (a) Opens up or down
   (b) Vertex
   (c) \(p\)-value
   (d) Focus
   (e) Equation of directrix

For the following problems, it may be helpful to sketch the parabola on graph paper.

5. Write the equation of a vertical parabola whose vertex is (-1,4) and \(p\)-value is 3.

6. Write the equation of a parabola whose vertex is (3,9) and focus is (3,6).

7. Write the equation of a parabola whose vertex is (0,1) and directrix is \(y = -2\).

8. Write the equation of a parabola whose directrix is \(y = -1\) and focus is (2,-5).
Exercise #1: Write the general equation of a parabola that has a vertical axis of symmetry.

Exercise #2: Write the general equation of a parabola that has a horizontal axis of symmetry.

Exercise #3: For each of the following equations of a parabola (a) state whether it opens right, left, up, or down and identify the (b) vertex, (c) p-value, (d) focus, and (e) the equation of the directrix.

1. \( x = \frac{1}{4} (y - 2)^2 + 3 \)
   (a) Opens
   (b) Vertex
   (c) p-value
   (d) Focus
   (e) Equation of directrix

2. \( x - 5 = -\frac{1}{16} (y + 3)^2 \)
   (a) Opens
   (b) Vertex
   (c) p-value
   (d) Focus
   (e) Equation of directrix
3. \[4(x + 6) = y^2\]
   (a) Opens
   (b) Vertex
   (c) p-value
   (d) Focus
   (e) Equation of directrix

4. \[(x - 1)^2 = 8y - 24\]
   (a) Opens
   (b) Vertex
   (c) p-value
   (d) Focus
   (e) Equation of directrix

5. Write an equation for the set of points which are equidistant from the origin and the line \(x = -2\).

6. Write the equation of a parabola whose focus is \((3, 1)\) and directrix is \(y = 5\).

7. Write an equation for the set of points which are equidistant from \((4, -2)\) and the line \(y = 4\).
LESSON #49 – FOCUS AND DIRECTRIX OF A PARABOLA – PART 2
COMMON CORE ALGEBRA II HOMEWORK

For each of the following equations of a parabola (a) state whether it opens right, left, up, or down and identify the (b) vertex, (c) p-value, (d) focus, and (e) the equation of the directrix.

8. \( x = \frac{1}{12} (y + 3)^2 - 2 \)
   (a) Opens
   (b) Vertex
   (c) p-value
   (d) Focus
   (e) Equation of directrix

9. \( x - 1 = -\frac{1}{20} (y + 3)^2 \)
   (a) Opens
   (b) Vertex
   (c) p-value
   (d) Focus
   (e) Equation of directrix

10. \( (x + 1)^2 = y + 1 \)
    (a) Opens
    (b) Vertex
    (c) p-value
    (d) Focus
    (e) Equation of directrix
11. \((y - 3)^2 = 8(x - 5)\)
   
   (a) Opens
   
   (b) Vertex
   
   (c) p-value
   
   (d) Focus
   
   (e) Equation of directrix

12. Write the equation of the parabola with focus (1,6) and directrix \(x = 10\).

13. Write an equation for the set of points which are equidistant from (0,2) and the x-axis.

14. Write the equation of the parabola with focus (-2,0) and directrix the y-axis.

15. Write the equation for the set of points equidistant from (2,3) and \(y = -3\)